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ABSTRACT

An understanding of limits is valuable for learning advanced mathematics. Fundamental to almost all branches of mathematics, studying and applying limits enable students to pursue meanings instead of calculations to many different applications. The idea of nearness to a point instead of exactness is different for many students. Using spreadsheets and other forms of technology provide a variety of ways for the learner to understand and determine limits. This paper describes the use of spreadsheets and spreadsheet macros to calculate limits and visualize output. A variety of examples are shown for differentiation and integration. Through the use of technology, relevant situations become the focus of mathematical study. Concepts are more important than computations. And technology enables the student to be an active participant in the learning of mathematics. (Author)

Learning the Concepts of Limits with Technology

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Abstract

An understanding of limits is valuable for learning advanced mathematics. Fundamental to almost all branches of mathematics, studying and applying limits enable students to pursue meanings instead of calculations to many different applications. The idea of nearness to a point instead of exactness is different for many students. Using spreadsheets and other forms of technology provide a variety of ways for the learner to understand and determine limits. This paper describes the use of spreadsheets and spreadsheet macros to calculate limits and visualize output. A variety of examples are shown for differentiation and integration. Through the use of technology, relevant situations become the focus of mathematical study. Concepts are more important than computations. And technology enables the student to be an active participant in the learning of mathematics.

Introduction

Learning how to find limits is difficult for many students. In order to understand the concept of limit the learner has to evaluate a function around a chosen point for many values. Often limits cannot be found analytically. Although learning how to use technology might also interfere with learning, technology helps develop an understanding and an intuition for finding limits in more than one way. This paper demonstrates ways of calculating limits with spreadsheets. Spreadsheets are easy to learn and concepts are enhanced for the learner. Not only calculations but visualizations of limits are easily presented.

Finding limits is valuable in many areas of calculus including differentiation, integration, and continuity. The study of limits is also fundamental to understanding topics such as sequences, probability, and fractal geometry. Increasing insights into function behavior near a point and at infinity are possible with technology.

A Spreadsheet as a Technological Tool

Many technological tools and software such as DERIVE, MATHEMATICA, or MATLAB, and programmable calculators are helpful for studying mathematics. But these technologies allow the learner to push buttons without an understanding of what is happening when performing calculations. Difficulty in learning the meaning to what is being calculated may occur. With a spreadsheet, every step is clearly illustrated. Techniques are applied as learned and no black box exists for doing mathematics. Numerous benefits of using spreadsheets are known. These include: a) They are widely available, b) Macros are easy to construct, and c) Graphs and tables are easily made without any knowledge of complex programming languages.

A Spreadsheet Template for Limits

A spreadsheet template that calculates the left-sided and right-sided limit for any function is presented in Table 1. In LOTUS 1-2-3 for WINDOWS, using the Chart XY Type Graphs commands shows the limit relationships.

Insert Table 1 about here

Spreadsheet Macros for Limits

Macros constructed on a spreadsheet are small programs that implement a number of calculations or steps similar to FORTRAN, Basic or C programming languages. Making macros for automating calculations can be easily done in LOTUS 1-2-3 for WINDOWS. Using the sequence of Tools Macro Run commands runs the macro starting at the indicated range. (Also by pushing \. the classic LOTUS 1-2-3 for DOS can be used) Another way to develop a macro is to construct a macro button. This is done by a) using Tools Draw Button and then highlighting the sequence of steps necessary for running a macro. Clicking on the name in the macro button box, the macro is named. Clicking on this button will run the macro automatically. The Help menu in WINDOWS is a great aid in constructing spreadsheet macros.

In Table 2, a macro for calculating the left-sided limits for a function is displayed. The macro waits for the user to press the ENTER key for each calculation.

Insert Table 2 about here

This macro calculates the value of a function as x approaches a specific value a from the left. Each cell carries out a specific task. They are:

Cell e2 (line 1) erases the range of cells from a1 to d15

Cell e3 (line 2) displays the statement Enter the value x is approaching, a

Cell e4 (line 3) waits for the user to enter a value

Cell e5 (line 4) displays the statement Enter the number of terms desired.

Cell e6 (line 5) waits for the user to enter the number of terms in cell a5.

Cell e7 (line 6) places the value in cell a5 in cell a6 as a counter

Cell e8 (line 7) uses a for loop with 1 as the starting value, ending with a6-1, counting by ones and loop starts in cell e9

Cell e9 (line 8) states is h in b7. (h is a sequence in terms of a6 converging to 0, can be faster or slower as needed)

Cell e10 (line 9) h is calculated

Cell e11 (line 10) displays approaching from the left in cell b9

Cell e12 (line 11) displays a-h in terms of a3 and a7

Cell e13 (line 12) requests user to press ENTER for each calculation

Cell e14 (line 13) displays values of function in terms of cell a9 at each a-h value

Cell e15 (line 14) waits for the press of the ENTER key

Pressing the ENTER key continues the loop until the desired number of terms is reached. A macro can be placed anywhere on a spreadsheet. For convenience the macro described is placed near the output. Cell formulas do not have to be known because the macro automates calculations. Any desired function can be entered in cell e14 in terms of a9. Cell e10 enters a sequence for h in terms of a6. This sequence for h must converge to 0 and can be made faster or slower by changing cell a6.

Calculating Limits with Spreadsheets

Examples of limits are available in many calculus textbooks (Larson, Hostetler & Edwards, 1998, Urso, 1995 and Lial, Hungerford and Miller, 1999). In order to illustrate the spreadsheet template and macro defined in Table 1 and Table 2, the following examples are provided for evaluating limits.

Example 1: Calculating A Limit

Find the limit for: $\lim_{x \to 5.1} (6.1-x)^{(3/(5.1-x))}$

The left-sided and right-sided limit for the example are shown in Table 3. Graph 1 and Graph 2 display output visually. The results of the calculations using the macro defined in Table 2 are presented in Table 4.

Insert Table 3 About Here

Insert Graph 1 About Here

Insert Graph 2 About Here

Insert Table 4 About Here

Relevant situations become the focus of learning with the use of technology. The output of a practical situation applied to population growth (Example 2) is displayed in Table 5 and Table 6.

Example 2: Differentiation Application:

A researcher claims that the population of a certain community is growing over x years in thousands and given by : $f(x) = \frac{66x}{x^2 + 12} + 87.$

Some questions that could be considered for the classroom are:

a) As the years increase the population grows to how many people? (87000)

b) Find the rate of change for population growth in 1 year. (4.295858)

c) Find the rate of change for population growth in 5 years. (-0.626735)

Part a is answered by using Table 1 or Table 2. When the number of years is large the population grows to 87000. In order to calculate the rate of change for a function, the limiting process, numerical differentiation is applied. A spreadsheet for numerical differentiation is given in Table 5. The macro output for the rate of change (derivative) of population growth when the time is 5 years is shown in Table 6.

Insert Table 5 About Here

Insert Table 6 About Here

This application is a good example of how limit concepts are useful for limiting values and differentiation. Without using technology, the user should estimate that for large values of x, the function approaches 87. Calculating the rate of change of the population with respect to time at various times is found by using the limit definition for the derivative at a point a:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

Finding the first derivative can be found by applying the spreadsheet and macro shown in Table 1 and Table 2. In Table 2, changing cell e3 to {goto}b3~Enter the value for a~, cell e11 to {goto}b9~a+h term~, and cell e12 to {goto}a9~+a3+a7~ enable the user to compute numerical derivatives. In cell e14 the user enters the expression for (f(a+h)-f(a))/h in terms of cells a3, a9, and a11.

Calculating definite integrals using limits of lower and upper approximating Riemann Sums is illustrated with Example 3. Finding definite integrals are valuable for calculating areas, probability, expectation values, average values and many other concepts.

Example 3: Integration Application

The amount of soda placed in a can by a filling machine is approximated by a normal distribution with a mean of 12.1 ounces and a standard deviation of .06 ounce. Find the probability that a can of soda will contain between 12 and 12.12 ounces.

There are a variety of ways of finding this probability. Applying DERIVE, spreadsheets, normal curve tables and the built-in command @NORMAL are a few. The area under the normal probability distribution between a and b is given by $P(a \leq x \leq b) = \frac{1}{\sigma\sqrt{2\pi}} \int e^{-(x-\mu)^2/2\sigma^2}dx$, where μ is the mean of the population, σ is the standard deviation of the population and x is the value of interest. Although this integral cannot be evaluated exactly, approximating the result by Riemann

Sums, Simpson's Rule or the Trapezoidal Rule are good approaches. In Example 3, a is 12 ounces and b is 12.12 ounces. Using Riemann Sums to approximate integrals is another concept that can be learned by studying limits. More accurate results can be found using Simpson's Rule or the Trapezoidal Rule. (Larson, Hostetler, & Edwards, 1998)

Table 7 displays output for Example 3 by calculating lower approximating sums. Using the macro in Table 8 for calculating lower approximating sums gives a result of 0.582713 after 5000 iterations or subintervals. Of course using the built-in command @NORMAL provides a more accurate result. The format of the @NORMAL command is @NORMAL(value of interest, mean, standard deviation).

Applying @NORMAL(12.12,12.1,0.06)-@NORMAL(12,12.1,0.06) gives a result of 0.582768.

Insert Table 7 About Here

Insert Table 8 About Here

Cautious Interpretation of Output

Output that occurs with the use of technology must be carefully examined. Technology may provide incorrect results. Or the user may not understand or interpret results correctly. When this happens, the user should ask why or what is wrong. The use of technology provides solutions in more innovative and effective ways, but an understanding of the reasonableness of an answer is necessary. In Table 9, an example of what may happen when using the macro defined in Table 2 is shown. When choosing a large number of terms or choosing a faster converging sequence for h, (for example, using 0.1^n instead of $1/n^3$) incorrect results occur.

Insert Table 9 About Here

Many students rely too much on technology without any assessment of the accuracy of the output as shown in Table 9. With different sequences for h or a large number of terms, trouble with output occurs. Having students determine whether and why a result is incorrect is important. In Table 9, the reason why a10 is 0 is because the limit becomes (5-5)/1.0E-18. Results like these indicate that the user cannot always assume computer output is correct.

Advantages and Disadvantages of Technology Use in the Classroom

There are advantages and disadvantages to the use of technology in mathematics learning. Using spreadsheets makes learning more interactive. Students explore and study more complex examples and exercises. Relevant applications are more intuitive and easily studied. Symbol manipulation is not the only method of studying limits. Visualizing and approximating outcomes are techniques that become more easily implemented.

Some of the disadvantages of technology use are known. For instance, computers are not accessible and are not portable for many students. From my classroom experiences, numerous students have difficulty in applying the order of operations for computing limits. Also learning how to use technology places an additional burden on learning mathematics.

Conclusion

Using spreadsheets for finding limits is a great motivator for learning. Spreadsheet macros provide a more effective method for calculating a great number of values without filling the spreadsheet with too much output. Studying the behavior of a function around a point is more meaningful through the use of technology. Visualizing ideas is valuable to showing the behavior of a function as x approaches a particular value. Spreadsheets are great tools for doing a lot of mathematics without doing a lot of programming.

Students using spreadsheets learn more than limit concepts. They learn that a) Spreadsheets are easy to use; b) More exercises can be studied; c) Mathematics is relevant and practical; d) Order of operations and how functions are input are important; e) Visualization is helpful in understanding the concept of limits; f) There are many ways to study and apply limits; g) Conjecture is helpful in determining the correctness of results; h) Functions in 2 and 3 variables can be explored; i) Interpretation of output is crucial to understanding concepts; and j) Connections can be made across a number of mathematical fields.

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Table 1

Calculating Limits Cell Formulas

	A	B	C	D	E	F
1	f(x) =					
2	a =					
3	n	h	a-h		f(a-h)	
4	1	1/a4^3	b2-b4		f(+c4)	
5	2	1/a5^3	b2-b5		f(+c5)	
6	3	1/a6^3	b2-b6		f(+c6)	
7	4	1/a7^3	b2-b7		f(+c7)	
16	13	1/a16^3	b2-b15		f(+c16)	
17	14	1/a17^3	b2-b16		f(+c17)	

Note. The f(a-h) columns displays the function values as x approaches a from the left. For calculations of right sided limits create an a+h column and an f(a+h) column. Any number of iterations can be found by using the COPY and PASTE commands. In cell b1 document the function to be evaluated. In cell b2 enter the value of a. In column b, h is a sequence converging to 0. Cells can be identified with large or small letters.

Table 2.

Calculating Left-Sided Limits By Macro

	A	B	E	F
2			/real.d15~	
3			{goto}b3~Enter the value x is approaching, a~	
4			{goto}a3~{?}~	
5			{goto}b5~Enter the number of terms desired~	
6			{goto}a5~{?}~	
7			{goto}a6~+a5~	
8			{for a6,1,(a6-1),1,e9}~	
9			{goto}b7~is h ~	
10			{goto}a7~1/a6^3~	
11			{goto}b9~approaching from left~	
12			{goto}a9~+a3-a7~	
13			{goto}b11~Press ENTER for each calculation~	
14			{let a11, (6.1-a9)^(3/(5.1-a9))}~	
15			{goto}a11~{?}~	

Note. For a right-sided limit replace cell e12 with {goto}b9~approaching a from the right.and cell e13 with {goto}a9~+a3+a7~. In cell e10, h is converging to 0.

Table 3

Example 1 Calculated on a Spreadsheet

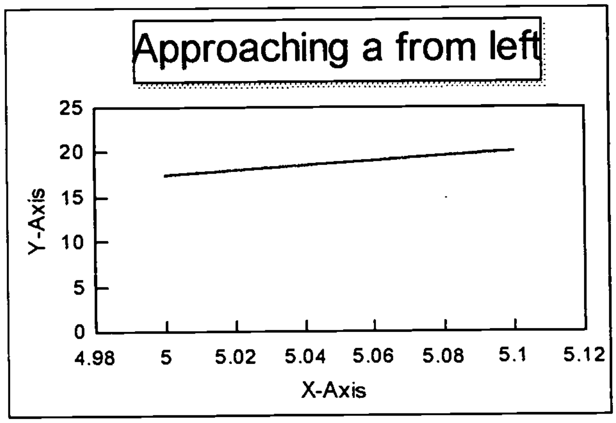
	A	B	C	D	E	F
1	f(x) =	(6.1-x)^(3/(5.1-x))				
2	a =	5.1				
3	n	h	a-h	f(a-h)	a+h	f(a+h)
4	1	1	4.1	8	6.1	ERR
5	2	0.125	4.975	16.8912	5.225	24.64942
6	3	0.037037	5.062963	19.02548	5.137037	21.26296
7	4	0.015625	5.084375	19.62499	5.115625	20.56693
8	5	0.008	5.092	19.84721	5.108	20.32932
9	6	0.00463	5.09537	19.94696	5.10463	20.22594
10	7	0.002915	5.097085	19.99806	5.102915	20.17374
11	8	0.001953	5.098047	20.02685	5.101953	20.14454
12	9	0.001372	5.09868	20.04429	5.101372	20.12695
16	13	0.000455	5..099545	20.07183	5.100455	20.09926
17	14	0.000364	5.099636	20.07456	5.100364	20.09652

Note. The correct limit is e^3 or 20.8554. This result is found after 126 calculations with $h = 1/n^3$.

Cell f4 shows ERR because the computer is raising 0 to a power.

Graph 1

Left-Sided Limit Visualization of Example 1.



Note. Visualizing left-sided limits is achieved by graphing columns c and d of Table 3 as x and y. Visualizing right-sided limits is performed similarly. For column d, enter the desired function in terms of column c.

Graph 2

Right-Sided Limit Visualization of Example 1.

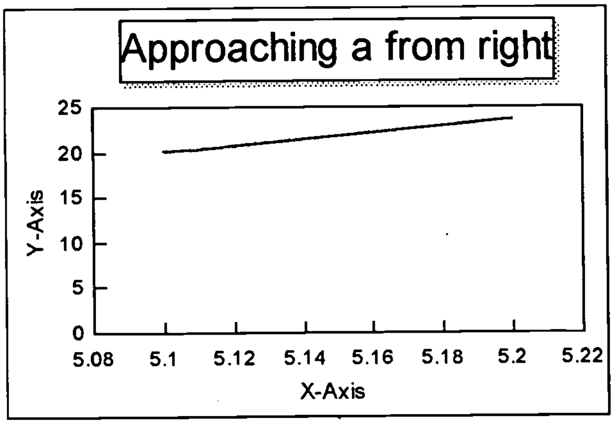


Table 4

Macro Output for Example 1

	A	B	C	D	E	F
2						
3	5.1	Enter the value x is approaching, a				
5	100	Enter the number of terms desired				
6	100					
7	1.0E-06	is h				
8						
9	5.099999	approaching a from left				
10						
11	20.8551	Press ENTER for each calculation				

Note. Cell a6 counts the number of calculations for each limit.

Table 5

Differentiation Application Calculated on a Spreadsheet

1	A	B	C	D	E	F
2						
3	Enter the function here		f'(x) is evaluated at			
4	f(x) =		a =			
5		(66x/(x^2+12))+87	5			
6	n	h	a+h	f(a+h)	f(a)	(f(a+h)-f(a))/h
7	1	1	6	95.25	95.91892	-0.66892
8	2	0.125	5.125	95.83953	95.91892	-0.63514
9	3	0.037037	5.037037	95.89561	95.91892	-0.62934
10	4	0.015625	5.015625	95.90911	95.91892	-0.62785
11	5	0.008	5.008	95.9139	95.91892	-0.62731
12	6	0.00463	5.00463	95.91602	95.91892	-0.62707
20	14	0.000364	5.000364	95.91869	95.91892	-0.62676
21	15	0.000296	5.000296	95.91873	95.91892	-0.62676

Note. The correct result by analytic methods is -0.62673. After 77 calculations the result is reached on the spreadsheet with h = $1/n^3$. In order to document the function and the value of the derivative at a, calculations start at cell a7.

Table 6

Differentiation Macro Output for Application

	A	B	C
1			
2			
3	5	Enter the value for a	
4			
5	100	Enter the number of terms desired	
6	100		
7	1.0E-06	is h	
8			
9	5.000001	a+h term	
10	-.62673	Press ENTER for each calculation	

Note. Changing the sequence h provides results that appear incorrect. See Table 9.

Table 7

Integration Example Calculated on a Spreadsheet

	A	B	C	D	E	F
1						
3	Enter the mean		12.1			
5	Enter the standard deviation		0.06			
7	Enter the lower endpoint a		12.0			
9	Enter the upper endpoint b		12.12			
11	Enter the number of subdivisions					
12		n =	100			
14	h = (b-a)/n					
15	h = 0.0012					
16	Constant = 1/(square root of 2 pi)					
	iteration i	Left Hand	Function	Sums	Sums times Constant	1
21	1	12	0.000299	0.000299	0.00199	
22	2	12.0012	0.000309	0.000609	0.004046	
23	3	12.0024	0.000320	0.000928	0.006171	
24	4	12.0036	0.000330	0.001258	0.008366	
25	5	12.0048	0.000341	0.001599	0.010632	
26	6	12.0060	.000352	0.001950	0.012971	

Table 7... continued...

119	99	12.1176	0.001149	0.86085	0.572383
120	100	12.12	0.001135	0.087228	0.57998

Note. After 100 iterations the result is 0.57998. The first cell b21 formula for the column titled Left Hand starts with c7+(a21-1)*b15 from the definition for Lower Riemann Sums. The first cell c21 formula for the function column starts with the formula @EXP(-(b21-c3)^2/(2*c5^2))*b15. This formula calculates an approximation for normal probabilities. Using COPY and PASTE can generate any number of iterations desired. The sums are found by using @SUM command.

Table 8.

Integration Macro

	D	E
1		
3		/rea3.d21~
4		{goto}b3~Enter the mean~
5		{goto}a3~{?}~
6		{goto}b4~Enter the standard deviation~
7		{goto}a4~{?}~
8		{goto}b6~Enter the lower endpoint a~
9		{goto}a6~{?}~
10		{goto}b7~Enter the upper endpoint b~
11		{goto}a7~{?}~
12		{goto}b9~Enter the number of subdivisions~
13		{goto}a9~{?}~
14		{goto}b11~is the value for h~
15		{let a11, (a7-a6)/a9)~
16		{let a16, 0}
17		{goto}a10~+a9~
18		{for 10,1,(a10-1), 1, e19}~

Table 8...continued...

19	{let a14, +a6+(+a10-1)*+a11}~
20	{let a16, +a16+@exp(-(a14-a3)^2/(2*a4^2))*a11}~
21	{let a17, +a16*1/(@sqrt(2*@pi)*a4)}~
22	{goto}a18~Lower Sums~

Note. The ouput for Example 3 using this macro with a = 12 and b = 12.12 is 0.57998 using 100 subdivisions and 0.582713 using 5000 subdivisions. Upper Approximating Sums can be calculated by changing cell e19 to {let a14, +a6+a10*a11}~.

Table 9.

Interpretation of Output

	A	B	C
1			
2			
3	5	Enter the value for a	
4			
5	20	Enter the number of terms desired	
6	18		
7	1.0E-18	is h	
8			
9	5	a+h term	
10	0	Press ENTER for each calculation	

Note. In Table 9, cell a10 is incorrect because (5-5)/1.0E-18 is calculated as 0.

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